Eur. Phys. J. C 55, 85–93 (2008)
DOI 10.1140/epjc/s10052-008-0565-y

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article – Theoretical Physics

Renormalized Wick expansion for a modified PQCD

Alejandro Cabo Montes de Oca[1,2,a]

[1] Group of Theoretical Physics, Instituto de Cibernética, Matemática y Física, Calle E, No. 309, Vedado, La Habana, Cuba
[2] The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy

Received: 1 September 2007 /
Published online: 11 March 2008 –

Abstract. The renormalization scheme for the Wick expansion of a modified version of the perturbative QCD introduced in previous works is discussed. Massless QCD is considered by implementing the usual multiplicative scaling of the gluon and quark wave functions and vertices. However, also massive quark and gluon counterterms are allowed in this massless theory since the condensates are expected to generate masses. A natural set of expansion parameters of the physical quantities is introduced: the coupling itself and the two masses m_q and m_g associated to quarks and gluons, respectively. This procedure allows one to implement a dimensional transmutation effect through these new mass scales. A general expression for the new generating functional in terms of the mass parameters m_q and m_g is obtained in terms of integrals over arbitrary but constant gluon or quark fields in each case. Further, the one loop potential is evaluated in more detail in the case when only the quark condensate is retained. This lowest order result again indicates the dynamical generation of quark condensates in the vacuum.

1 Introduction

The relevance of properly understanding QCD is difficult to overestimate. However, due to the known difficulties associated to its strong interaction properties, the physical predictions of QCD are also extremely far from a satisfactory understanding. Thus, the investigation of the properties of the theory should be considered from all possible angles, as it has been done for many years [1–10]. The motivations of this work (and of previous works of various authors [11–19]) can be summarized as follows: firstly, let us recall that in modern high energy physics, the masses are normally searched assuming they would appear as generated by a spontaneous or dynamical symmetry breaking in theories taken to be massless at the start. Then, the circumstance that the QCD conveys the strongest force of Nature, in combination with the fact that the massless version of the theory does not have a definite mass parameter, directly leads to the physical relevance of examining the scales of dynamical mass allowed by symmetry breaking processes in massless QCD [7, 20]. In former works, we have obtained indications about the possibility of generating masses in QCD [13, 19]. In [13], modified Feynman rules were employed for evaluating the quark masses from the Dyson equation in which the simplest corrections to the self-energy determined by the condensates were retained. For this purpose the gluon condensate parameter C_g was evaluated by fixing the mean value of the gluon Lagrangian (a quantity which, in the proposed picture, is non-vanishing in the lowest approximation) to its estimated value in the literature. The result for initially massless quarks surprisingly gave a value of one third of the proton mass [13]. That is, a prediction of the constituent quark masses followed.

Motivated by this result, in [19] we considered similar evaluations assuming also the presence of a quark condensate for any flavor C_f, $f = 1, 2, ..., 6$ and for the gluons C_g in massless QCD. In this case, by properly selecting the coefficients C_f and C_g, it was possible to obtain the quark masses as singularities of the propagator for the six quarks, also in the simplest approximation. Thus, the question emerged of the possibility for those condensate values to be generated as a result of dynamical symmetry breaking in massless QCD. In [15] this issue began to be considered by evaluating particular summations of one loop diagrams. The results were positive in the direction of supporting the generation of the gluon as well as the quark condensates. However, the instability of the potential at zero value, in spite of indicating this dynamical creation of the quark condensate, did not have a form bounded from below. This fact did not allow us to determine a definite mean value of the condensate to be approached in the system after stabilization. However, this result can be a consequence of the first evaluation of a scheme in which the renormalization had not yet been implemented or might be produced by the low order corrections that were calculated.

Therefore, in this work we start considering the renormalization of the proposed expansion and its application to evaluate the first order contributions to the effective action in terms of the quark and gluon condensates. For this pur-

[a] e-mail: cabo@icmf.inf.cu

pose, we are already supported by the discussion in [14] in which the gauge invariance of the proposed Feynman expansion was argued for. Also, and importantly, a procedure was also devised in that paper for eliminating the singularities appearing in the graph expansion due to the presence of Dirac delta functions of the momenta [14,21].

Here we start by introducing the renormalization prescription in the Euclidean version of the generating function of the Green functions $Z = \exp(W)$ depending on external sources, which will also be a function of the condensate parameters for quarks and gluons. Multiplicative renormalization is implemented for the fields and coupling constants in the Wick expansion. However, it should be noticed that mass counterterms will also be added, although the bare theory before the adiabatic connection of the interaction is being assumed massless QCD. This assumption is essential: we are considering that the mass counterterms will automatically be generated by the interactions, although they cannot be implemented by the multiplicative scaling of the fields and parameters. The effective action is then introduced as usual, as the Legendre transform over the external sources in favor of the mean values of the quantum fields. The effective potential determined by it is also a function of the gluon and quark condensates and naturally it should show a minimum with respect to these quantities at the ground state. Here, the field values are fixed to vanish in this ground state assuming the Lorentz invariance of the vacuum from the beginning. All the generating functionals defined are considered as expanded in power series of the coupling constant g, and the two defined mass parameters $m_q = (g^2 C_q)^{\frac{1}{3}}$ and $m_g = (g^2 C_g)^{\frac{1}{2}}$. This reordering of the expansion of physical quantities allows one to implement the dimensional transmutation effect in the considered massless QCD. The generating functional Z is obtained in a form in which the effects of the condensates are represented by Gaussian weighted averages over constant and homogeneous background gauge fields for gluons and quarks, respectively. These formulae are expected to be considered for detailed calculations elsewhere. Finally, the lowest order correction to the potential is evaluated in more detail for the case of the single presence of a quark condensate. The dependence of the potential in the lowest order indicates a tendency to the dynamical generation of this condensate.

The work will proceed as follows. In Sect. 2, the Feynman expansion for QCD in Euclidean variables and the conventions to be used are described. Next, the renormalized generating functional expressing the Feynman expansion in momentum space is written. Section 3 considers the calculation of the effective potential in zero order in g for the case of the single presence of the quark condensate. Finally, the results are reviewed and commented on in the summary.

2 Generating functional $Z\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]$

The purpose of this section is to present a renormalized version of the modified perturbative expansion under consideration. With this aim the starting action for massless QCD in Euclidean variables will be taken to be in the form

$$S = \int \mathrm{d}x \left(-\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \frac{1}{2\alpha} \partial_\mu A^a_\mu \partial_\nu A^a_\nu - \overline{\Psi^i}_q \mathrm{i}\gamma_\mu D^{ij}_\mu \Psi^j_q - \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right), \tag{1}$$

where the field intensity and covariant derivatives follow the conventions

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu , \\ D^{ij}_\mu &= \partial_\mu \delta^{ij} + \mathrm{i} g A^a_\mu T^{ij}_a , \quad D^{ab}_\mu = \partial_\mu \delta^{ab} + g f^{abc} A^c_\mu , \\ \{\gamma_\mu, \gamma_\nu\} &= -2\delta_{\mu\nu}, \quad [T_a T_b] = \mathrm{i} f^{abc} T_c . \end{aligned} \tag{2}$$

The Fourier decomposition for any field, i.e. the gauge one, will be assumed to be in the form

$$\begin{aligned} A^b_\mu(x) &= \int \frac{\mathrm{d}k}{(2\pi)^D} \exp(\mathrm{i}k_\mu x_\mu) A^b_\mu(k) , \\ A^b_\mu(k) &= \int \mathrm{d}x \exp(-\mathrm{i}k_\mu x_\mu) A^b_\mu(x) . \end{aligned}$$

Then the renormalized Green's function, generating functional including the gluon and quark condensate parameters C_q and C_g as discussed in [14], is expressed as follows:

$$\begin{aligned} Z\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right] &= \frac{I\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]}{I[0,0,0,0]} , \\ I\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right] &= \exp\left(V^{\mathrm{int}} \left[\frac{\delta}{\delta j}, \frac{\delta}{\delta \overline{\eta}}, \frac{\delta}{-\delta\eta}, \frac{\delta}{\delta\overline{\xi}} \frac{\delta}{-\delta\xi} \right] \right) \\ &\times \exp\left(\int \frac{\mathrm{d}k}{(2\pi)^D} j(-k) \frac{1}{2} D(k) j(k) \right) \\ &\times \exp\left(\int \frac{\mathrm{d}k}{(2\pi)^D} \overline{\eta}(-k) G_q(k) \eta(k) \right) \\ &\times \exp\left(\int \frac{\mathrm{d}k}{(2\pi)^D} \overline{\xi}(-k) G_{gh}(k) \xi(k) \right) , \end{aligned} \tag{3}$$

in which the only changes with respect to the functional associated to the usual perturbative QCD appear in only two of the three free propagators of the expansion, the quark and the gluon ones [11–19]:

$$\begin{aligned} D^{ab}_{\mu\nu}(k) &= \delta^{ab} \left(\frac{1}{k^2} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \theta_N(k) + C^b_g \delta^D(k) \delta_{\mu\nu} \right) , \\ G^{ij}_q(k) &= \delta^{ij} \left(\frac{\theta_N(k)}{m_f + \gamma_\mu k_\mu} + C^b_q \delta^D(k) I \right) , \\ G^{ab}_{gh}(k) &= \delta^{ab} \frac{\theta_N(k)}{k^2} . \end{aligned} \tag{4}$$

That is, the quark and gluon propagators now include the condensation effects through the C_q and C_g parameters, respectively. However, an important additional change is also present in (4). The usual Feynman propagators are regularized in the neighborhood of zero momentum by multiplying them by

$$\theta_N(k) = \theta(\sigma - |k|) , \quad |k| = (k_\mu k_\mu)^{\frac{1}{2}} , \tag{5}$$

where θ is the Heaviside function and σ is an infinitesimal momentum cutoff, which we will call the Nakanishi parameter. As discussed in [14], this regularization naturally arises when gauge theory propagators are properly defined. Here we assumed that after passing to the Euclidean version of the theory, this prescription can be translated to eliminate a neighborhood of the momentum space near the zero momentum point. This infrared regularization in combination with a dimensional regularization rule for products of Dirac delta functions evaluated at zero momenta allowed one in [14] to eliminate the singularities appearing in the perturbative series due to the presence of products of delta functions evaluated at zero momenta and of products with propagators also being evaluated at vanishing momenta.

The vertices defining the interaction in the Wick expansion formula (3) have the decomposition

$$\begin{aligned} V^{\mathrm{int}} &= V_g^{(1)} + V_g^{(2)} + V_q^{(1)} + V_{gh}^{(1)} \\ &\quad + (Z_1-1)V_g^{(1)} + (Z_4-1)V_g^{(2)} + (Z_{1F}-1)V_q^{(1)} \\ &\quad + (\tilde{Z}_1-1)V_{gh}^{(1)} + V_g^{(0)} + V_q^{(0)} + V_{gh}^{(0)}\,, \end{aligned} \tag{6}$$

where the superindices indicate the order of the coupling constant associated to the original vertices in the bare action. As usual the unrenormalized bare fields (indicated with a superindex b) will be related to their renormalized counterparts through the factors 'Z' as

$$\begin{aligned} A_\mu^b &= Z_3^{\frac{1}{2}} A_\mu\,, \\ \Psi_q^b &= Z_2^{\frac{1}{2}} \Psi_q\,, \quad \overline{\Psi}_q^b = Z_2^{\frac{1}{2}} \overline{\Psi}_q\,, \\ \chi^b &= Z_3^{\frac{1}{2}} \chi\,, \quad \overline{\chi}^b = Z_3^{\frac{1}{2}} \overline{\chi}\,, \end{aligned} \tag{7}$$

with the correspondence, also usual, between the sources:

$$\begin{aligned} j_\mu^b &= Z_3^{-\frac{1}{2}} j_\mu\,, \\ \eta_q^b &= Z_2^{-\frac{1}{2}} \eta_q\,, \quad \overline{\eta}_q^b = Z_2^{-\frac{1}{2}} \overline{\eta}_q\,, \\ \xi^b &= Z_3^{-\frac{1}{2}} \xi, \quad \overline{\xi}^b = Z_3^{-\frac{1}{2}} \overline{\xi}\,. \end{aligned} \tag{8}$$

It should be underlined that the bare condensate parameters C_q^b and C_g^b appear in the free propagators because these constants had not been expanded yet in their renormalized and counterterm contributions. This will be performed later within a more convenient representation for this purpose. It is also assumed that the scale parameter μ of the dimensional regularization links the dimensional coupling g with its dimensionless value g_0 as follows:

$$g = g_0 \mu^{2-\frac{D}{2}} = g_0 \mu^{\epsilon}\,. \tag{9}$$

The expressions for each of the vertices entering V^{int} in (6) are given in the appendix. Let us examine more closely the vertex $V_q^{(0)}$ associated to the mass and wave-function renormalization of the bare theory. It should be first recalled that the modified expansion under consideration was constructed by deriving the Wick expansion for massless QCD, in which the mass parameter is absent. Then the renormalization procedure is assumed to represent the physics of an adiabatic connection of the interaction from an originally massless theory. Therefore, we estimate as the most natural procedure to fix the bare masses of gluons and quarks as taking them vanishing. However, it is clear that since the theory has been argued to generate mass [13, 19], the connection of the interaction should be expected to produce mass counterterms in the renormalized action. These terms will be assumed to appear among the quark counterterm vertices $V_q^{(0)}$ in Appendix with the form

$$\begin{aligned} V_q^{(0)}\left[\frac{\delta}{\delta\overline{\eta}}, \frac{\delta}{-\delta\eta}\right] &= \int\int \mathrm{d}k_1\,\mathrm{d}k_2 (2\pi)^D \delta^D(k_1+k_2) \\ &\quad \times \frac{\delta}{\delta\eta(k_2)}\left((Z_2-1)k_{1\mu}\gamma_\mu + \delta m(g_0, m_q, m_g)\right)\frac{\delta}{\delta\overline{\eta}(k_1)}\,, \end{aligned}$$

although a non-vanishing δm means a break of pure multiplicative renormalization. This does not seem to be a complication since multiplicative renormalization is known to be broken when the theory has no symmetries that enforce the vanishing of allowed counterterms having no counterpart in the original bare action.

Now, the source terms in the gluon condensate propagator can be represented as a Gaussian integral over constant and homogeneous gauge boson fields as follows:

$$\begin{aligned} &\exp\left[\left(\int \frac{\mathrm{d}k}{(2\pi)^D} j_\mu^a(-k) C_g^b \delta(k) j_\mu^a(k)\right)\right] \\ &= \exp\left(\frac{C_g^b}{(2\pi)^D} j_\mu^a(0) j_\mu^a(0)\right) \\ &= \frac{1}{(2\pi)^{(N^2-1)D}\mathcal{N}_g(Z_3)} \int \mathrm{d}\alpha_\mu^a \exp\left[\left(\frac{Z_3-1}{Z_3}\right)\frac{\alpha_\mu^a\alpha_\mu^a}{2}\right] \\ &\quad \times \exp\left[-\frac{\alpha_\mu^a\alpha_\mu^a}{2} + \left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}} j_\mu^a(0)\alpha_\mu^a\right] \\ &= \frac{1}{(2\pi)^{(N^2-1)D}\mathcal{N}_g(Z_3)} \\ &\quad \times \exp\left[\frac{(Z_3-1)}{2}\left(\frac{2C_g}{(2\pi)^D}\right)^{-1}\frac{\partial^2}{2\partial j_\mu^a \partial j_\mu^a}\right] \\ &\quad \times \int \mathrm{d}\alpha_\mu^a \exp\left(-\frac{\alpha_\mu^a\alpha_\mu^a}{2} + \left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}} j_\mu^a(0)\alpha_\mu^a\right), \end{aligned} \tag{10}$$

where $\mathcal{N}_g$ is a normalization constant that cancels with a similar one appearing in the normalizing factor $I[0,0,0,0]$ in (3). Here, the process of, say, "translating" the renormalization part $(Z_3-1)C_g$ of the gluon condensate to the counterterms is started by expressing the terms containing those parts as an exponential of a quadratic form in the derivatives over the sources.

For quarks, an analogous formula in terms of interaction over anti-commuting fermion fields will be employed.

It has the form

$$\exp\left(\int \frac{\mathrm{d}k}{(2\pi)^D}\overline{\eta}^i_u(-k)C^b_q\delta(k)\eta^i_u(k)\right)$$
$$=\exp\left(\frac{C^b_q}{(2\pi)^D}\overline{\eta}^i_u(0)\eta^i_u(0)\right)$$
$$=\frac{1}{\mathcal{N}_q(Z_2)}\int \mathrm{d}\overline{\chi}^i_u\,\mathrm{d}\chi^i_u \exp\left[\frac{(Z_2-1)}{Z_2}\overline{\chi}^i_u\chi^i_u\right]$$
$$\exp\left[-\overline{\chi}^i_u\chi^i_u+\left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}}\left(\overline{\eta}^i_u(0)\chi^i_u+\overline{\chi}^i_u\eta^i_u(0)\right)\right],$$
$$=\frac{1}{\mathcal{N}_q(Z_2)}\exp\left[\frac{(Z_2-1)}{Z_2}\left(\frac{C_q}{(2\pi)^D}\right)^{-1}\frac{\partial^2}{-\partial\eta^i_u(0)\partial\overline{\eta}^i_u(0)}\right]$$
$$\times\int \mathrm{d}\overline{\chi}^i_u\mathrm{d}\chi^i_u \exp\left(-\frac{\overline{\chi}^i_u\chi^i_u}{2}+\left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}}\right.$$
$$\left.\times\left(\overline{\eta}^i_u(0)\chi^i_u+\overline{\chi}^i_u\eta^i_u(0)\right)\right), \qquad (11)$$

in which again the appearing factor $\mathcal{N}_q$ will be cancelled by a similar one appearing in $I[0,0,0,0]$. In the above expressions the condensate parameters have naturally been decomposed in the following way:

$$C^b_g = Z_3C_g = (Z_3-1)C_g + C_g\,,$$
$$C^b_q = Z_2C_q = (Z_2-1)C_q + C_q\,.$$

It can be recalled that the condensates were created by the action over the vacuum of the exponential of quadratic forms for the bare gluon and quark creation operators [12, 14]. Therefore, the renormalization of these operators implies that the bare parameters should be related to the renormalized ones through the same Z_3 or Z_2 constants for gluons and quarks, respectively. It can be noted that quadratic terms in the ghost fields also appeared within the quadratic form defining the gluon condensate. However, since the ghosts have the same renormalization constant as the gluons, the Z_3 proportionality between the bare and renormalized gluon condensates should remain valid.

The following relationships help to transform the functional derivatives over the sources in terms of the usual derivatives over the zero momentum components of these sources, when integrated around zero momentum within the Nakanishi neighborhood:

$$\int \mathrm{d}k\frac{\delta}{\delta j(k)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\theta_N(k)=\frac{\partial}{\partial j(0)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right],$$
$$\int \mathrm{d}k\frac{\delta}{\delta\eta(k)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\theta_N(k)=\frac{\partial}{\partial\eta(0)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right],$$
$$\int \mathrm{d}k\frac{\delta}{\delta\overline{\eta}(k)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\theta_N(k)=\frac{\partial}{\partial\overline{\eta}(0)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right],$$
$$\int \mathrm{d}k\frac{\delta}{\delta\xi(k)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\theta_N(k)=0\,,$$
$$\int \mathrm{d}k\frac{\delta}{\delta\overline{\xi}(k)}F\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\theta_N(k)=0\,.$$

After employing the above relations, the Z functional can be written in a form in which the effects of the condensates are represented by the newly incorporated gluon and quark constant and homogeneous fields α, $\overline{\chi}$ and χ (below, they will be called *auxiliary* fields). The expression for Z is

$$Z\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]=\frac{I\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]}{I[0,0,0,0]},$$
$$I\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]=\frac{1}{\mathcal{N}}\int\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\mathrm{d}\chi\exp\left[-\overline{\chi}^i_u\chi^i_u-\frac{\alpha^a_\mu\alpha^a_\mu}{2}\right]$$
$$\times\exp\left[\widehat{V}^{\mathrm{int}}\left[\frac{\delta}{\delta j}+\left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}}\alpha,\frac{\delta}{\delta\overline{\eta}}+\left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}}\chi,\right.\right.$$
$$\left.\left.\frac{\delta}{-\delta\eta}\left(\frac{C_q}{(2\pi)^D}\right)^{\frac{1}{2}}\overline{\chi},\frac{\delta}{\delta\overline{\xi}}\frac{\delta}{-\delta\xi},\alpha,\overline{\chi},\chi\right]\right]$$
$$\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}j(-k)\frac{1}{2}D^F(k)j(k)\right]$$
$$\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}\overline{\eta}(-k)G^F_q(k)\eta(k)\right]$$
$$\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}\overline{\xi}(-k)G^F_{gh}(k)\xi(k)\right], \qquad (12)$$

in which $\mathcal{N}$ is a normalization constant that again cancels in the ratio of I functions in (3). Note that now the propagators are the usual massless Feynman ones, but the vertex terms $\widehat{V}^{\mathrm{int}}$ include additional contributions associated to the renormalization of the condensate parameters. The vertices expressed in terms of the derivatives over the sources have the form

$$\widehat{V}^{\mathrm{int}}\left[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\xi}},\frac{\delta}{-\delta\xi},\frac{\partial}{\partial j(0)},\frac{\partial}{-\partial\eta(0)},\frac{\partial}{\partial\overline{\eta}(0)}\right]$$
$$=V^{\mathrm{int}}\left[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\xi}}\frac{\delta}{-\delta\xi}\right]$$
$$+\frac{(Z_3-1)}{Z_3}\left(\frac{2C_g}{(2\pi)^D}\right)^{-1}\frac{\partial^2}{2\partial j^a_\mu(0)\partial j^a_\mu(0)}$$
$$+\frac{(Z_2-1)}{Z_2}\left(\frac{C_q}{(2\pi)^D}\right)^{-1}\frac{\partial^2}{-\partial\eta^i_u(0)\partial\overline{\eta}^i_u(0)}\,.$$

Let us search, in what follows, for a re-ordering of the perturbative expansion seeking to explicitly introduce the dimensional transmutation effect in the modified representation [20].

For this purpose a first idea comes from the fact that the auxiliary fields enter as kinds of background constant fields. This fact directly leads to a proposal similar to the one made in [11]. In that work, the modified propagators considered were first introduced in order to be employed in the modifying of the perturbative expansion. However, there was also discussed an alternative scheme, in which the generating functional Z for QCD was considered as an average over constant gluon fields. This superposition allowed one to argue that the Fradkin's general functional differential equations for Z [22]

should be exactly obeyed by the mean value over constant fields of the auxiliary Z functionals associated to arbitrary constant mean fields. As will be seen from the following discussion, the final form of the functional obtained here indicates the equivalence between the two proposals advanced in [11]. However, in that work there was no clarity about how to conveniently define the weighted average.

The auxiliary fields now appear in the vertices constant gluon or quark background fields. Therefore, it seems natural to express the expansion (before the mean value over the auxiliary quantities is taken) in terms of the gluon and quark propagators in the presence of such fields.

For this purpose, from $\widehat{V}^{\text{int}}$ in (12) the contribution to its expansion, coming from the terms being second order in the gluon and quark fields, but also including auxiliary backgrounds, will be subtracted. These terms can now be acted on by the exponential containing the usual Feynman propagators contracted with the sources. Further, the recourse can be employed of expressing back the exponential of the quadratic forms of the sources in terms of the Feynman propagators in (12), as a continuous integral over the gluon, quark and ghost fields. Then the previously mentioned exponential of the quadratic form in the functional derivatives over the gluon and quark fields simply will produce an additional quadratic form within the exponential of the continuous integral. Collecting the total quadratic form in the fields leads to a new modified free path integral over which the remaining exponential of the vertices will act. Its expression is

$$
\begin{aligned}
&Z^{(0)}\left[j,\eta,\overline{\eta},\xi,\overline{\xi}|C_q^b,C_g^b\right] \\
&= \frac{1}{\mathcal{N}}\int\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\,\mathrm{d}\chi \exp\left[-\overline{\chi}_u^i\chi_u^i - \frac{\alpha_\mu^a\alpha_\mu^a}{2}\right] \\
&\times \int \mathcal{D}\left[A,\overline{\Psi},\Psi,\overline{c},c\right] \\
&\times \exp\Bigg[-\int \frac{\mathrm{d}k}{(2\pi)^D}\frac{1}{2}A_\mu^a(-k)\Bigg(\mathbf{D}^{ac}\mathbf{D}^{cb}\delta_{\mu\nu} \\
&- \frac{\mathbf{D}_\mu^{ac}\mathbf{D}_\nu^{cb}+\mathbf{D}_\nu^{ac}\mathbf{D}_\mu^{cb}}{2} + \frac{\delta^{ab}}{\alpha}k_\mu k_\nu\Bigg)A_\nu^b(k) \\
&+ \int \frac{\mathrm{d}k}{(2\pi)^D}\overline{c}_\mu^a(-k)k_\mu \mathbf{D}_\mu^{ac}c(k) \\
&+ \int \frac{\mathrm{d}k}{(2\pi)^D}\overline{\Psi}^i(-k)\gamma_\mu \mathbf{D}_\mu^{ij}\,\Psi^j(k) \\
&+ \int \frac{\mathrm{d}k}{(2\pi)^D}\overline{\Psi}^{i,u}(-k)(-g)\left(\frac{C_q^b}{(2\pi)^D}\right)^{\frac{1}{2}}\gamma_\mu^{uv}T_a^{ij}\,\chi^{j,v}A_\nu^a(k) \\
&+ \int \frac{\mathrm{d}k}{(2\pi)^D}A_\mu^a(-k)\overline{\chi}^{i,u}(-g)\left(\frac{C_q^b}{(2\pi)^D}\right)^{\frac{1}{2}}\gamma_\mu^{uv}T_a^{ij}\,\Psi^{j,v}(k) \\
&+ \int \frac{\mathrm{d}k}{(2\pi)^D}\overline{c}^a k^2 c^b + \int \frac{\mathrm{d}k}{(2\pi)^D}(j(-k)A(k)+\overline{\eta}(-k)\Psi(k) \\
&+\overline{\Psi}(k)\eta(-k)+\overline{\xi}(-k)c(k)+\overline{c}(-k)\xi(k))\Bigg],
\end{aligned}
$$

where $\mathbf{D}_\mu^{ij} = k_\mu\delta^{ij} + g\left(\frac{2Cg}{(2\pi)^D}\right)^{\frac{1}{2}} T_a^{ij}\alpha_\mu^a$ and $\mathbf{D}_\mu^{ab} = k_\mu\delta^{ij} - \mathrm{i}g\left(\frac{2Cg}{(2\pi)^D}\right)^{\frac{1}{2}} f^{abc}\alpha_\mu^c$.

The above expression can be converted into a more compact form by defining a composite field and its source and their conjugates, having boson and fermion components, as follows:

$$
\Phi = \left\{\begin{array}{c} A_\mu^a \\ \Psi^{r,u} \\ c^a \end{array}\right\}, \quad \Phi^* = \overbrace{\underbrace{A_\mu^a \;\; \overline{\Psi}^{r,u} \;\; \overline{c}^a}}, \tag{13}
$$

$$
J = \left\{\begin{array}{c} j_\mu^a/2 \\ \eta^{r,u} \\ \xi^a \end{array}\right\}, \quad J^* = \overbrace{\underbrace{j_\mu^a/2 \;\; \overline{\eta}^{r,u} \;\; \overline{\xi}^a}}. \tag{14}
$$

Therefore, a new free generating functional $Z^{(0)}$ can be expressed in the following way:

$$
\begin{aligned}
&Z^{(0)}\left[j,\eta,\overline{\eta},\xi,\overline{\xi}|C_q^b,C_g^b\right] \\
&= \frac{1}{\mathcal{N}}\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\,\mathrm{d}\chi \exp\left[-\overline{\chi}_u^i\chi_u^i - \frac{\alpha_\mu^a\alpha_\mu^a}{2}\right] \\
&\times \exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\times \int \mathcal{D}[\Phi]\exp\Bigg[\int \frac{\mathrm{d}k}{(2\pi)^D}\Phi^*(-k)S^{-1}(k)\Phi(k) \\
&\qquad + J^*(-k)\Phi(k)+\Phi^*(-k)J(k)\Bigg] \qquad (15) \\
&= \frac{1}{\mathcal{N}}\int\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\,\mathrm{d}\chi \exp\left[-\overline{\chi}_u^i\chi_u^i - \frac{\alpha_\mu^a\alpha_\mu^a}{2}\right] \\
&\times \exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\times \exp\left[\int \frac{\mathrm{d}k}{(2\pi)^D}\Phi^*(-k)S^{-1}(k)\Phi(k)\right] \\
&\times \exp\left[J^*(-k)S(k)J(k)\right] \qquad (16) \\
&= \frac{1}{\mathcal{N}}\int\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\,\mathrm{d}\chi \exp\left[-\overline{\chi}_u^i\chi_u^i - \frac{\alpha_\mu^a\alpha_\mu^a}{2}\right]\mathcal{M}(\alpha,\overline{\chi},\chi) \\
&\times \exp\left[\int \frac{\mathrm{d}k}{(2\pi)^D}J^*(-k)S(k)J(k)\right]. \qquad (17)
\end{aligned}
$$

The quantity $F_{\mu\nu}^a(\alpha) = gf^{abc}\alpha_\mu^b\alpha_\nu^c$ is the field intensity of the gluon constant field α of which the Lagrangian appeared due to the shift done in the auxiliary fields. Also, the functional integral differential has been written as

$$
\mathcal{D}[\Phi] = \mathcal{D}[A,\overline{\Psi},\Psi,\overline{c},c],
$$

and the matrix S^{-1} has the block structure

$$
S^{-1} = \left\{\begin{array}{ccc} \mathbf{A}/2 & \mathbf{C} & 0 \\ \mathbf{D} & \mathbf{B} & 0 \\ 0 & 0 & \mathbf{G} \end{array}\right\}, \tag{18}
$$

where the matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{G}$ are defined by the expressions

$$\mathbf{A}^{(\mu,a),(\nu,b)}(\alpha) \equiv -\left(\mathbf{D}^{ac}\mathbf{D}^{cb}\delta_{\mu\nu} - \frac{\mathbf{D}_\mu^{ac}\mathbf{D}_\nu^{cb}+\mathbf{D}_\nu^{ac}\mathbf{D}_\mu^{cb}}{2} + \frac{\delta^{ab}}{\alpha}k_\mu k_\nu\right),$$

$$\mathbf{B}^{(u,r),(v,s)}(\alpha,\overline{\chi},\chi) \equiv \gamma_\mu\left(k_\mu\delta^{ij}+g\beta_\mu^a T_a^{ij}\right),$$

$$\mathbf{D}^{(u,r),(\nu,b)}(\overline{\chi},\chi) \equiv -g\left(\frac{C_q^b}{(2\pi)^D}\right)^{\frac{1}{2}}\gamma_v^{uq}T_b^{ij}\chi^{q,t},$$

$$\mathbf{C}^{(\mu,a),(v,s)}(\overline{\chi},\chi) \equiv -g\left(\frac{C_q^b}{(2\pi)^D}\right)^{\frac{1}{2}}\overline{\chi}^{q,t}\gamma_\mu^q v T_b^{ts},$$

$$\mathbf{G}^{ab}(\alpha) = k_\mu \mathbf{D}_\mu^{ab},$$

$$\mathbf{D}_\mu^{ab} = k_\mu\delta^{ab} + gf^{abc}\beta_\mu^c,$$

$$\beta_\mu^a = \left(\frac{2C_g^b}{(2\pi)^D}\right)^{\frac{1}{2}}\alpha_\mu^a.$$

In (16) the function of the auxiliary fields $\mathcal{M}$ is given by

$$\begin{aligned}
\mathcal{M}(\alpha,\overline{\chi},\chi) &= \frac{1}{\mathcal{N}}\exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\quad\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}\Phi^*(-k)S^{-1}(k)\Phi(k)\right] \\
&= \frac{1}{\mathcal{N}}\exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\quad\times\int\mathcal{D}\left[A,\overline{\Psi},\Psi,\overline{c},c\right] \\
&\quad\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}\left(A(-k)\frac{\mathbf{A}(k)}{2}A(k)\right.\right. \\
&\quad+\overline{\Psi}(-k)\mathbf{B}(k)\Psi(k)+A(-k)\mathbf{C}(k)\Psi(k) \\
&\quad\left.\left.+\overline{\Psi}(-k)\mathbf{D}(k)A(k)+\overline{c}(-k)\mathbf{G}(k)c(k)\right)\right] \\
&= \frac{1}{\mathcal{N}}\exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\quad\times\det{}^{-\frac{1}{2}}[\mathbf{A}]\det\left[\mathbf{B}-\mathbf{D}\mathbf{A}^{-1}\mathbf{C}\right]\det[\mathbf{G}] \\
&= \frac{1}{\mathcal{N}}\exp\left[-\frac{1}{4}V^D F_{\mu\nu}^a(\alpha)F_{\mu\nu}^a(\alpha)\right] \\
&\quad\times\exp\left[-\frac{1}{2}\mathrm{Tr}[\log[\mathbf{A}]]\right. \\
&\quad\left.+\mathrm{Tr}\left[\log[\mathbf{B}-\mathbf{D}\mathbf{A}^{-1}\mathbf{C}]+\mathrm{Tr}[\log[\mathbf{G}]]\right]\right],
\end{aligned}$$

in which the matrix indices of the fields with the block matrices have not been written explicitly in order to avoid a more cumbersome expression. However, their incorporation seems to be clearly feasible. Henceforth, the following expression can be written for the generating functional Z:

$$Z[j,\eta,\overline{\eta},\xi,\overline{\xi}] = \frac{I[j,\eta,\overline{\eta},\xi,\overline{\xi}]}{I[0,0,0,0]}, \tag{19}$$

$$\begin{aligned}
I\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right] &= \frac{1}{\mathcal{N}}\int\int\mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\,\mathrm{d}\chi\exp\left[-\overline{\chi}_u^i\chi_u^i-\frac{\alpha_\mu^a\alpha_\mu^a}{2}\right] \\
&\quad\times\mathcal{M}(\alpha,\overline{\chi},\chi) \\
&\quad\times\exp\left[\tilde{V}^{\mathrm{int}}\left[\frac{\delta}{\delta j},\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta\overline{\xi}}\frac{\delta}{-\delta\xi},\alpha,\overline{\chi},\chi\right]\right] \\
&\quad\times\exp\left[\int\frac{\mathrm{d}k}{(2\pi)^D}J^*(-k)S(k)J(k)\right],
\end{aligned} \tag{20}$$

in which the composite sources J and propagator S are defined in (13) and (18). The vertex terms $\tilde{V}^{\mathrm{int}}$ in (19) are equal to the ones in $\widehat{V}^{\mathrm{int}}$ plus one additional second order in g, which is linear in the gluon auxiliary field. Its expression is given at the end of Appendix .

2.1 Expansion parameters

At this point it is useful to recall that the modified perturbative expansion under consideration has a set of three parameters on which the physical quantities depend: (g, C_q and C_g). All of them will be assumed here to have a dimension defined in powers of the renormalization scale μ. However, the alternative representation (19) suggests a modification of the relevant parameters for the expansion when seeking for the realization of the dimensional transmutation effect [20]. This idea comes from the form of the propagator S in (19). It can be noted that this new free propagator can be made gauge coupling independent simply by defining the new set of independent parameters:

$$\begin{aligned}
g &= g, \\
m_g^2 &= g^2 C_g, \\
m_q^2 &= \left(g^2 C_q\right)^{\frac{2}{3}}.
\end{aligned} \tag{21}$$

With this definition, the propagator of the composite field becomes coupling independent and all the vertices not associated to counterterms are mass independent. It can be remarked that these gluon and quark mass parameters were the ones defining the prediction for the constituent masses in [13, 19].

It seems useful to resum here the dimensions of the various fields and constants:

$$\begin{aligned}
D[A] &= \frac{D-2}{2}, \quad D[\Psi] = \frac{D-1}{2} = D\left[\overline{\Psi}\right], \\
D[\chi] &= \frac{D-2}{2} = D\left[\overline{\chi}\right], \quad D[g] = 2-\frac{D}{2}, \\
D[C_g] &= D-2, \quad D[C_q] = D-1.
\end{aligned}$$

Then the quark mass parameter m_q having the expression $m_q^2 = (g^2 C_q)^{\frac{2}{3}}$ has a dimension equal to two, not changing under the regularization, since

$$D\left[m_q^2\right] = \frac{2}{3}(4-D+D-1) = 2.$$

The same is valid for the gluon mass parameter, of which the dimension is

$$D\left[m_g^2\right] = (4-D+D-2) = 2.$$

2.2 Connected Green functions generator and effective action

The generating functional W of the connected Green functions and the effective action Γ are defined now by the usual Legendre transformation by

$$W\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]=\log\left[Z\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]\right],$$
$$\Gamma\left[A,\overline{\Psi},\Psi,\overline{\chi},\chi\right]=W\left[j,\eta,\overline{\eta},\xi,\overline{\xi}\right]-\int \mathrm{d}x\left(j\,A+\overline{\eta}\Psi+\overline{\Psi}\eta+\overline{\xi}\chi+\overline{\chi}\xi\right),$$

in which the mean fields are determined from Z through

$$A_\mu^a(x)=\frac{\delta}{\delta j_\mu^a(x)}\log\left(Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]\right),$$
$$\Psi(x)=\frac{\delta}{\delta\overline{\eta}(x)}\log\left(Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]\right),$$
$$\overline{\Psi}(x)=\frac{\delta}{\delta(-\eta(x))}\log\left(Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]\right),$$
$$c(x)=\frac{\delta}{\delta\overline{\xi}(x)}\log\left(Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]\right),$$
$$\overline{c}(x)=\frac{\delta}{\delta(-\xi(x))}\log\left(Z[j,\eta,\overline{\eta},\xi,\overline{\xi}]\right).$$

It is important to notice that by the definition of the original Z, W is exactly the sum of all connected diagrams, in which the lines are the addition of the usual Feynman propagators plus the "condensate propagators". However, after the introduction of the auxiliary fields, the alternative form of Z became a mean value of generating functionals $Z(\alpha,\overline{\chi},\chi)=\exp(W(\alpha,\overline{\chi},\chi))$ depending on the auxiliary fields. It is suspected that the mean value of $Z(\alpha,\overline{\chi},\chi)$ coincides with the exponentiation of the average of $W(\alpha,\overline{\chi},\chi)$, and therefore this quantity should be equal to W. However, we do not have the proof of this property yet. It will be considered in a future extension of the work, since some hints point in the direction of its validity, at least approximately in the infinite volume limit.

3 Quark effective potential in order g^0

Let us consider the one loop effective action when only the quark condensate is retained. Then, Γ at zero values of the mean fields and their sources can be written in the form

$$\Gamma(m_q)=\log\left[\frac{Z[0,0,0,0,0|C_q,0]}{Z[0,0,0,0,0|0,0]}\right]$$
$$=\log\left[\int\int \mathrm{d}\overline{\chi}D\chi\exp\left(-\overline{\chi}\chi\right)\exp\left\{V^{(D)}\times\int\frac{\mathrm{d}k}{(2\pi)^D}\mathrm{Tr}_{\mathrm{spin,color}}\left(\log\left[(k_\mu\gamma_\mu)^{(u,r),(v,s)}-\frac{m_q^3}{(2\pi)^D}\gamma_\mu^{uq}T_a^{rt}\chi^{q,t}\frac{1}{(-k^2)}\overline{\chi}^{q',t'}\gamma_\mu^{q'v}T_a^{t's}\right]-\log\left[(k_\mu\gamma_\mu)^{(u,r),(v,s)}\right]\right)\right\}\right].$$

The mean value over the auxiliary field appearing above can be expressed as follows:

$$\mathcal{G}=\int\int \mathrm{d}\overline{\chi}\mathrm{d}\chi\exp(-\overline{\chi}\chi)\times\exp\left\{V^D\int\frac{\mathrm{d}q}{(2\pi)^D}\mathrm{Tr}_{\mathrm{spin,color}}\left(\log\left[(q_\mu\gamma_\mu)^{(u,r),(v,s)}-\gamma_\mu^{uq}T_a^{rt}\chi^{q,t}\frac{1}{(-q^2)}\overline{\chi}^{q',t'}\gamma_\mu^{q'v}T_a^{t's}\right]-\log\left[(q_\mu\gamma_\mu)^{(u,r),(v,s)}\right]\right)\right\}$$
$$=\int\int d\overline{\chi}d\chi\exp(-\overline{\chi}\chi)\exp\left\{V^D\left(\frac{m_q^3}{(2\pi)^D}\right)^{\frac{D}{3}}\mathcal{F}(D)\right\}.$$

The factor $\mathcal{F}(D)$ is only dependent on the dimension, and for this pure quark case it is convergent due to the high dimension of the parameter m_q^3. Its expression in terms of an integral over the dimensionless variables takes the form

$$\mathcal{F}(D)=\int\int \mathrm{d}\overline{\chi}\mathrm{d}\chi\exp(-\overline{\chi}\chi)\times\exp\left\{\int\frac{\mathrm{d}q}{(2\pi)^D}\mathrm{Tr}_{\mathrm{spin,color}}\left(\log\left[(q_\mu\gamma_\mu)^{(u,r),(v,s)}-\gamma_\mu^{uq}T_a^{rt}\chi^{q,t}\frac{1}{(-q^2)}\overline{\chi}^{q',t'}\gamma_\mu^{q'v}T_a^{t's}\right]-\log\left[(q_\mu\gamma_\mu)^{(u,r),(v,s)}\right]\right)\right\}. \quad (22)$$

It seems possible to evaluate $\mathcal{G}$ if the expansion over the auxiliary fermion fields is properly investigated. However, this requires a separate detailed study to be considered elsewhere. Here, we will evaluate it, in a kind of mean field approximation, in which the product of the fields $\chi^{q,t}\overline{\chi}^{q',t'}$ in (22) will be replaced by its "mean" value over the integration of the auxiliary fields. That is,

$$\chi^{q,t}\overline{\chi}^{q',t'}\to\int\int \mathrm{d}\alpha\,\mathrm{d}\overline{\chi}\mathrm{d}\chi\exp(-\overline{\chi}\chi)\chi^{q,t}\overline{\chi}^{q',t'}=\delta^{qq'}\delta^{tt'}.$$

After the use of the relations

$$T_a^{ik}T_a^{kj}=C_F\delta^{ij},\quad C_F=\frac{N^2-1}{2N},$$
$$\gamma_\mu^{uq}\gamma_\mu^{qv}=-D\delta^{uv},$$

the $\mathcal{F}(D)$ factor gets the simple form

$$\mathcal{F}(D)=\exp\left\{4N\int\frac{\mathrm{d}q}{(2\pi)^D}\log\left[1+\frac{4D^2(N^2-1)^2}{4N^2}\frac{1}{q^2}\right]\right\}$$
$$=\exp\left\{4N\frac{\pi^{\frac{D}{2}}D}{\Gamma\left(\frac{D}{2}+1\right)}\int\frac{q^{D-1}\mathrm{d}q}{(2\pi)^D}\times\log\left[1+\frac{4D^2(N^2-1)^2}{4N^2}\frac{1}{(q^2)^3}\right]\right\}.$$

Therefore, the potential obeys the expression

$$
\begin{aligned}
V(m_q) &= -\Gamma(m_q) = -V^D\left(\frac{g^2C_q}{(2\pi)^D}\right)^{\frac{D}{3}}\mathcal{F}(D) \\
&= -\frac{V^D}{(2\pi)^{\frac{D^2}{3}}}\mu^4\left(\frac{m_q}{\mu}\right)^D 4N\frac{\pi^{\frac{D}{2}}D}{\Gamma\left(\frac{D}{2}+1\right)} \\
&\quad\times\int\frac{q^{D-1}\,\mathrm{d}q}{(2\pi)^D}\log\left[1+\frac{4D^2(N^2-1)}{4N^2}\frac{1}{(q^2)^3}\right],
\end{aligned}
$$

which in the limit $D\to 4$ leads to the energy density $v(m_q)$:

$$
\begin{aligned}
v(m_q) &= \frac{V(m_q)}{V^4} \\
&= -\frac{8\pi^2 N m_q^4}{3(2\pi)^{\frac{16}{3}}}\int\frac{q^3\,\mathrm{d}q}{(2\pi)^4}\log\left[1+\frac{64(N^2-1)^2}{4N^2}\frac{1}{(q^2)^3}\right].
\end{aligned}
$$

The dependence of $v(m_q)$ on the mass parameter m_q is plotted in Fig. 1. As in [15], the result indicates dynamical generation of the quark condensate in this simple approximation. The result is unbounded from below. The possibility that higher order corrections could stabilize the minimum will be investigated in future extensions of this work.

4 Summary

We started here the investigation of the renormalization of the modified perturbation expansion for massless QCD proposed in previous works. A generating functional Z of the finite Green functions is constructed in terms of the renormalized coupling, parameters and fields. Mass counterterms are also introduced assuming their most probable needs, since the theory is expected to generate masses for the originally massless fields. However, the bare masses are assumed to vanish in consistency with the procedure of the connection of the interaction on a massless theory, which is associated to the starting of the unrenormalized generating functional. Expressions for the vertex terms are given. The Z functional is transformed to an alternative representation as a mean value over generating functionals associated to background field theories in the presence of homogeneous gluon and quark fields. In this variant of the formulation, the gluon and quark fields are coupled in a global propagator mixing the boson and fermion modes. The analysis suggests the convenience of introducing the coupling constant g as before, and the two mass parameters m_q and m_g as a new set of independent expansion parameters for the physical quantities. These parameters are simply related to the quark and gluon condensate constants C_q and C_g, respectively, and retain their dimension, equal to two, under dimensional regularization. Their relations to the constants reflecting the quark and gluon condensates are $m_q^2=(g^2C_q)^{\frac{2}{3}}$ and $m_g^2=g^2C_q$. It is an interesting outcome that one of those constants was relevant in the prediction of the constituent quark masses in [13, 15].

An evaluation of the lowest order contribution to the effective action is presented for the case in which only a quark condensate is retained. The renormalization at this modified tree level approximation was not required. The result indicates a dynamical generation of a quark condensate, a prediction that was also obtained in [15]. However, here it appears in a more systematic framework. For the extension of the work, we plan to investigate the possibilities that higher order contributions could produce a minimum of the potential. Such a result might open the way for the application of the modified expansion in justifying a kind of top condensate model as an effective description of massless QCD.

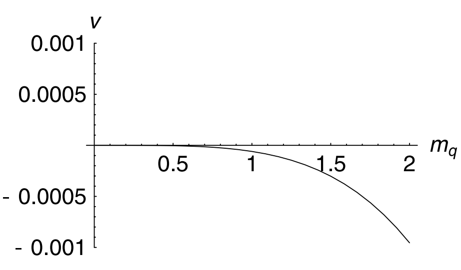

Fig. 1. The energy density estimated in the zeroth order in the coupling approximation, plotted as a function of the quark mass parameter m_q. Note that in this low order approximation, the result indicates a dynamical symmetry breaking under the generation of a quark condensate. The same outcome was obtained in [14] in which the same conclusion came from a less systematic analysis. The expansion parameters of any quantity are assumed to be the coupling constant g and the quark mass parameter m_q

Acknowledgements. The invitation and kind hospitality of the High Energy Section at the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy (ICTP) and its Head, S. Randjbar-Daemi, allowing a very helpful visit to the Centre, is deeply acknowledged. I also express my gratitude for support received from the Office of External Activities (OEA), through the Network on *Quantum Mechanics, Particles and Fields* (Net-35). The support to the work coming though the Grant CB03-2004 of the “Programa Nacional de Investigaciones Básicas en Matematica, Física y Ciencias de la Computación” has been also very much helpful. Useful remarks received from D. Treleani, G. Thompson, K. Narain are also very much appreciated.

Appendix

The explicit form of all the counterterms that appear in (6) is the following:

$$
\begin{aligned}
V_g^{(1)}\left[\frac{\delta}{\delta j}\right] &= \frac{1}{3!}\int\!\!\int \mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3(2\pi)^D\delta^D(k_1+k_2+k_3) \\
&\quad\times V^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(k_1,k_2,k_3)\frac{\delta}{\delta j^{a_1}_{\mu_1}(k_1)}\frac{\delta}{\delta j^{a_2}_{\mu_2}(k_2)}\frac{\delta}{\delta j^{a_3}_{\mu_3}(k_3)}, \\
V_g^{(2)}\left[\frac{\delta}{\delta j}\right] &= \frac{1}{4!}\int\!\!\int \mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3\,\mathrm{d}k_4(2\pi)^D \\
&\quad\times\delta^D(k_1+k_2+k_3+k_4)V^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4) \\
&\quad\times\frac{\delta}{\delta j^{a_1}_{\mu_1}(k_1)}\frac{\delta}{\delta j^{a_2}_{\mu_2}(k_2)}\frac{\delta}{\delta j^{a_3}_{\mu_3}(k_3)}\frac{\delta}{\delta j^{a_4}_{\mu_4}(k_4)}, \\
V_q^{(1)}\left[\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta},\frac{\delta}{\delta j}\right] &= \int\!\!\int \mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3(2\pi)^D
\end{aligned}
$$

$$\times \delta^D(k_1+k_2+k_3)\frac{\delta}{\delta\eta^{i_1}(k_1)}g\gamma_{\mu_3}T^{i_1 i_2}_{a_3}\frac{\delta}{\delta\overline{\eta}^{i_2}(k_2)}\frac{\delta}{\delta j^{a_3}_{\mu_3}(k_3)},$$

$$V^{(1)}_{gh}\left[\frac{\delta}{\delta\overline{\xi}},\frac{\delta}{-\delta\xi},\frac{\delta}{\delta j}\right]=\int\int \mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3(2\pi)^D$$

$$\times \delta^D(k_1+k_2+k_3)\frac{\delta}{\delta\eta^{a_2}(k_2)}(-\mathrm{i}g)k_{2\mu_3}f^{a_1 a_2 a_3}\frac{\delta}{\delta\overline{\xi}^{a_3}(k_3)}$$

$$\times\frac{\delta}{\delta j^{a_3}_{\mu_3}(k_3)},$$

$$V^{(0)}_{g}\left[\frac{\delta}{\delta j}\right]=\frac{1}{3!}\int\int \mathrm{d}k_1\,\mathrm{d}k_2(2\pi)^D\delta^D(k_1+k_2)$$

$$\times\frac{\delta}{\delta j^{a_2}_{\mu_2}(k_2)}\frac{(Z_3-1)}{2}\delta^{a_2 a_1}\left(k_1^2\delta_{\mu_2\mu_1}-k_{1\mu_2}k_{1\mu_1}\right)\frac{\delta}{\delta j^{a_1}_{\mu_1}(k_1)},$$

$$V^{(0)}_{q}\left[\frac{\delta}{\delta\overline{\eta}},\frac{\delta}{-\delta\eta}\right]=\int\int \mathrm{d}k_1\,\mathrm{d}k_2(2\pi)^D\delta^D(k_1+k_2)$$

$$\times\frac{\delta}{\delta\eta(k_2)}\left[(Z_2-1)k_{1\mu}\gamma_\mu+\delta m\left(g_0,C^0_q,C^0_g\right)\right]\frac{\delta}{\delta\overline{\eta}(k_1)},$$

$$V^{(0)}_{gh}\left[\frac{\delta}{\delta\overline{\xi}},\frac{\delta}{-\delta\xi}\right]=\int\int \mathrm{d}k_1\,\mathrm{d}k_2(2\pi)^D\delta^D(k_1+k_2)$$

$$\times\frac{\delta}{\delta\xi^{a_2}(k_2)}(Z_3-1)\delta^{a_2 a_1}k_1^2\frac{\delta}{\delta\overline{\xi}^{a_1}(k_1)}.$$

The only new vertex that appears in $\tilde{V}^{\mathrm{int}}$ in (19), after the quadratic form in the gluon and quark fields depending on the auxiliary fields is extracted in order to arrive at (19), has the form

$$V^{(2)}_{g}\left[\frac{\delta}{\delta j}\right]=\frac{1}{4!}\int\int \mathrm{d}k_1\,\mathrm{d}k_2\,\mathrm{d}k_3\,\mathrm{d}k_4(2\pi)^D$$

$$\times\delta^D(k_1+k_2+k_3+k_4)V^{a_1a_2a_3a_4}_{\mu_1\mu_2\mu_3\mu_4}(k_1,k_2,k_3,k_4)$$

$$\times\left(\frac{\delta}{\delta j^{a_1}_{\mu_1}(k_1)}+4\left(\frac{2C_g}{(2\pi)^D}\right)^{\frac{1}{2}}\alpha^a_{\mu_1}\delta^D(k_1)\right)$$

$$\times\frac{\delta}{\delta j^{a_2}_{\mu_2}(k_2)}\frac{\delta}{\delta j^{a_3}_{\mu_3}(k_3)}\frac{\delta}{\delta j^{a_4}_{\mu_4}(k_4)}.$$

References

1. E. Shuryak, The QCD vacuum, hadrons and superdense matter (World Scientific, Singapore, 1988)
2. T. Schafer, E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998)
3. H. Fritzsch, D. Holtmannspotter, Phys. Lett. B **338**, 290 (1994)
4. G.K. Savvidy, Phys. Lett. B **71**, 133 (1977)
5. I. Batalin, S.G. Matinyan, G.K. Savvidy, Sov. J. Nucl. Phys. **41**, 214 (1977)
6. J.M. Cornwall, R. Jackiw, E. Tomboulis, Phys. Rev. D **10**, 2428 (1974)
7. V.A. Miransky, in Nagoya Spring School on Dynamical Symmetry Breaking, ed. K. Yamawaki (World Scientific, Singapore, 1991)
8. J.M. Cornwall, Phys. Rev. D **26**, 1453 (1982)
9. W.A. Bardeen, C.T. Hill, M. Lindner, Phys. Rev. D **41**, 1647 (1990)
10. A. Cabo, O.K. Kalashnikov, A.E. Shabad, Nucl. Phys. B **185**, 473 (1981)
11. A. Cabo, S. Peñaranda, R. Martínez, Mod. Phys. Lett. A **10**, 2413 (1995)
12. M. Rigol, A. Cabo, Phys. Rev. D **62**, 074 018 (2000)
13. A. Cabo, M. Rigol, Eur. Phys. J. C **23**, 289 (2002)
14. A. Cabo, M. Rigol, Eur. Phys. J. C **47**, 95 (2006)
15. A. Cabo, D. Martinez-Pedrera, Eur. Phys. J. C **47**, 355 (2006)
16. P. Hoyer, NORDITA-2002-19 HE (2002), hep-ph/0203236
17. P. Hoyer, Proc. of the ICHEP 367–369, Amsterdam (2002), hep-ph/0209318
18. P. Hoyer, Acta Phys. Pol. B **34**, 3121 (2003) [hep-ph/0304022]
19. A. Cabo, JHEP **04**, 044 (2003)
20. S. Coleman, E. Weinberg, Phys. Rev. D **7**, 1888 (1973)
21. G. Leibbrandt, Rev. Mod. Phys. **47**, 849 (1975)
22. E.S. Fradkin, Quantum Field Theory and Hydrodynamics, Series Lebedev Physics Institute (Trudy Lebedev), Vol. 29 (Consultants Bureau, New York, 1967)